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CONTROLLING A TRANSONIC FLOW AROUND AIRFOILS BY MEANS OF ENERGY SUPPLY WITH ALLOWANCE FOR REAL PROPERTIES OF AIR

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The influence of molecular (thermodynamic and transport) properties of air on gas-dynamic effects of pulsed-periodic energy supply in a transonic flow around airfoils is studied. Relations for air with allowance for excitation of vibrations and dissociation are taken as the thermal equation of state and the caloric equation. The influence of the transport properties (viscosity) is taken into account approximately, within the framework of the boundary layer model. It is demonstrated that the effects in qualitative considerations do not depend on taking into account the molecular properties, but the allowance for internal degrees of freedom yields a significantly lower temperature than the temperature predicted by the ideal gas model. Allowance for viscosity ensures certain attenuation of the energy supply effects.

Key words: *transonic flow, airfoil, molecular properties of air, aerodynamic characteristics, energy supply.*

Introduction. Active external energy effects on the flow around various bodies in a wide range of flight velocities are intensely studied now [1–6]. In the present paper, we consider a transonic flow around airfoils. In [3–5], the gas-dynamic effects of pulsed-periodic energy supply were studied with the ideal gas model. The energy supply method considered in [3–5], however, involves a significant increase in temperature (up to several thousand degrees). At such a temperature and moderate pressure (fractions of the atmospheric pressure), vibrational motion of molecules and their dissociation occur; therefore, the ideal gas model becomes invalid.

At the same time, the gas-dynamic effects of energy supply, which were observed in [3], are so significant that they have to be studied with allowance for real thermodynamic and transport properties of air.

This problem with a constant local heat supply was studied in [6] by means of numerical modeling of a transonic flow around an airfoil with the use of the Reynolds-averaged Navier–Stokes equations (the specific heats were assumed to be constant). Starodubtsev [6] considered various aspects of the influence of heat supply on the flow around the airfoil and found an upstream shift of the closing shock wave, which confirmed the character of the flow structure reconstruction established previously (see, e.g., [3]). It should be noted that the position, compact shape, and power of the source of energy chosen in [6] did not increase the lift-to-drag ratio of the airfoil K. At the same time, the use of pulsed-periodic energy sources extended along the airfoil and adjacent to it [3–5] provided a substantial increase in the lift-to-drag ratio with the energy supply smaller than that in [6] by two orders of magnitude. In addition to the influence of volume heat supply, the effect of heat transfer between the flow and the body surface was also studied in the examined range of the free-stream Mach numbers (see, e.g., [7, 8]).

In the present work, we study the influence of pulsed-periodic energy supply on a transonic flow around a symmetric airfoil with allowance for real thermodynamic properties of air and also the effect of viscosity on aerodynamic characteristics of lifting airfoils in the boundary layer approximation with pulsed-periodic energy supply.

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Formulation of the Problem. The flow region is divided into the external inviscid flow domain and a thin boundary layer. The mathematical model used to describe a plane unsteady flow of an inviscid heat-non-conducting gas includes the Euler equations in a conservative form with a source term in the energy equation:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} + \frac{\partial \boldsymbol{G}}{\partial y} = \boldsymbol{Q},$$

$$\boldsymbol{U} = (\rho, \rho u, \rho v, e), \qquad \boldsymbol{F} = (\rho u, p + \rho u^2, \rho uv, u(p + e)), \tag{1}$$

$$\boldsymbol{G} = (\rho v, \rho uv, p + \rho v^2, v(p + e)), \qquad \boldsymbol{Q} = (0, 0, 0, q).$$

Here, the coordinate axes x and y are directed along the airfoil chord and normal to it, respectively; the parameters used for normalization are the chord length b for the coordinates, b/a_0 for the time t, a_0 for the gas velocity components u and v and the velocity of sound a, ρ_0 for the density ρ, $\rho_0 a_0^2$ for the pressure p and total energy of a unit volume of the gas e, and $\rho_0 a_0^3/b$ for the power q supplied to a unit volume of the gas; p_0 and a_0 are the dimensional free-stream pressure and velocity of sound; ρ_0 is found from the condition $p_0 = \rho_0 a_0^2$. In the case with pulsed-periodic energy supply, the value of q is determined by the expression

$$q = \Delta e\, f(t),$$

where $f(t) = \sum_i \delta(t - i\Delta t)$, $\delta(t)$ is the pulsed Dirac function, Δt is the energy supply period, and Δe is the energy supplied to a unit volume of the gas.

System (1) is supplemented with boundary conditions at the boundaries of the doubly connected computational domain Ω, which is a rectangle with the inner boundary corresponding to the contour of the airfoil considered. The conditions of an undisturbed flow are set on the left, upper, and lower boundaries of this domain, the "soft" conditions are imposed on the right boundary, and the no-slip condition is applied on the airfoil contour.

System (1) is solved numerically by the method described elsewhere (see, e.g., [4]).

In the model considered, pulsed energy supply is performed instantaneously; the gas density and velocity remain unchanged thereby. The gas energy density e in the energy supply zone increases by $\Delta e = \Delta E/\Delta S$ (ΔE is the total supplied energy normalized to $\rho_0 a_0^2 b^2$ and ΔS is the area of the energy supply zone). The energy is supplied in thin zones approximately shaped as rectangles adjacent to the airfoil. Significant nonlinear effects were observed in such a case in [3]. In particular, numerical experiments [3] with variations of the energy supply period Δt in the flow around the NACA-0012 airfoil at the Mach number $\mathrm{M}_\infty = 0.85$ showed that the shock-wave structure of the flow depends substantially on Δt. At high values of the parameter Δt, for instance, at $\Delta t = 0.5$, the flow structure is partly reconstructed; as a result, the closing shock is insignificantly shifted in the upstream direction. The position of the closing shock changes during the period. At $\Delta t = 0.05$, the shock stops ahead of the energy supply zone, and its position remains unchanged during the period. This value of Δt is considered as the limiting value.

In the ideal gas model with a constant ratio of specific heats γ, the following relations are valid:

$$p = (\gamma - 1)(e - \rho(u^2 + v^2)/2), \qquad a^2 = T = \gamma p/\rho.$$

Allowance for Real Thermodynamic Properties. Two models are used to take into account the real thermodynamic properties of air. In model No. 1, air is considered as an ideal mixture of O_2 and N_2 with constant values of the molar fractions x_m equal to 0.21 and 0.79, respectively. The rotational and vibrational degrees of freedom of molecules are described in the approximation that involves a rigid rotator and a harmonic oscillator with characteristic vibrational temperatures $T_{\nu,m} = 2228$ K and $T_{\nu,m} = 3336$ K for O_2 and N_2, respectively. In this model, the mean molar weight of the mixture is assumed to be constant, and the thermal equation of state has the same form as the equation for an ideal gas:

$$T = \gamma p/\rho.$$

Here, the temperature T is normalized to the free-stream temperature T_0, and γ is the ratio of specific heats for air (in our calculations, $\gamma = 1.4$). The specific enthalpy h is calculated by the formula

$$\gamma h = \frac{\gamma T}{\gamma - 1} + \sum_{m=1}^{2} \frac{x_m T_m}{\exp(T_m/T) - 1},$$

where $T_m = T_{\nu,m}/T_0$. The specific enthalpy and internal energy ε are related by the known expression $h = \varepsilon + p/\rho$. The velocity of sound is calculated by the formula

$$a^2 = \Big(\Big(\frac{\partial\rho}{\partial p}\Big)_T - \frac{T}{\rho^2}\Big(\frac{\partial\rho}{\partial T}\Big)_p^2 \Big/ \Big(\frac{\partial h}{\partial T}\Big)_p\Big)^{-1}.$$

This model of thermodynamic properties of air was used in [9].

In model No. 2, the real thermodynamic properties of air are taken into account by using analytical dependences of the specific enthalpy h and density ρ on the pressure p and temperature T [10]:

$$\rho = \rho(p,T), \qquad h = h(p,T).$$

These expressions are valid for $T = 200$–20,000 K and $p = 0.001$–1000 atm. The velocity of sound is found by the formula given above.

A comparison of the enthalpy, density, and velocity of sound determined by the formulas derived in [10] with the data obtained in [11–13] showed that the relative error of calculations was smaller than 3% for enthalpy and 1.5% for density in the ranges of temperatures and pressures indicated above; the error of calculating the velocity of sound was approximately 1% at $T = 200$–10,000 K. The formulas derived in [10] take into account vibrations of molecules, their dissociation, and ionization occurring once.

The initial distributions of parameters corresponding to a steady flow around the airfoil without energy supply were obtained for the variables ρ, u, v, and p with an absolute error of 10^{-4} in all grid nodes. The problem was solved as an unsteady problem from the beginning of energy supply until a periodic solution is obtained. The instant of obtaining a periodic solution was determined by comparing the averaged values of the airfoil drag coefficients in time intervals multiple to the energy supply period. The absolute error was smaller than 10^{-7}.

Boundary Layer. The flow around the airfoil occurs at high Reynolds numbers (about 10^5–10^7). In this flow regime, viscosity exerts a considerable effect only in a rather thin layer; therefore, it can be taken into account within the framework of the boundary layer model. The choice of the method was based on the known assumption about good agreement between the velocity distributions on the surface of a half-body and in an inviscid attached flow around an airfoil without indications concerning the airfoil shape [14]. Such a method of taking viscosity into account was used in solving inverse problems of aerohydrodynamics [15] and in solving problems of airfoil design and optimization by direct methods [16]. According to [15], the drag coefficient of the airfoil (with the wave drag being ignored) in the case of a viscous fluid with high Reynolds numbers can be approximately calculated by the Squire–Young formula

$$C_d = 2(v_k/v_0)^{3.2}\delta_k^{**}, \quad \delta_k^{**} = \delta_1^{**} + \delta_2^{**}, \quad \delta_j^{**} = v_k^n \nu^{1/(m+1)}\Big[\nu v_{tj}^{b-2}(\mathrm{Re}_{tj}^{**})^a + aA\Big|\int\limits_{\Delta_j}|v(\tau)|^{b-1}\Big|\,d\tau\Big]^{1/a}, \quad j = 1, 2,$$

where ν is the kinematic viscosity, v_k is the velocity, δ_k^{**} is the momentum thickness in a small vicinity of the end point of the airfoil, Δ_j is the length of the turbulent boundary layer on the upper and lower contours, and A, a, b, m, and n are empirical constants. The points of the transition of the laminar boundary layer to the turbulent state were chosen so that they corresponded to the point of the maximum velocity on the upper and lower contours. In this approach, one should take into account that, first, the calculation of the turbulent boundary layer characteristics yields overestimated values of momentum losses and, second, all methods of turbulent boundary layer calculations are based on empirical data corresponding to an incompressible fluid flow ($\mathrm{M} = 0$) and extrapolation of these data to the flow with $\mathrm{M} > 0$. Numerous calculations confirmed by experimental data show that calculations of the boundary layer on single airfoils (and on sets of airfoils) at $\mathrm{M} < 1.5$ can be performed with sufficient accuracy in the same manner as the incompressible fluid calculations if the velocity distribution corresponding to a compressible gas flow is used. For the above-indicated velocities, allowance for compressibility in boundary layer calculations leads to a decrease in the value of δ^{**} by several percent. As was seen in experiments, the friction stress at $\mathrm{M} < 1.5$ depends only weakly on the Mach number, and the momentum thickness is determined by the velocity distribution on the airfoil, which depends on the Mach number and, hence, on flow compressibility.

Calculation Results. The calculations with allowance for real thermodynamic properties of air were performed for the NACA-0012 airfoil at the angle of attack $\alpha = 0°$ in the flow with the Mach number $\mathrm{M}_\infty = 0.85$ with the limiting value of the period $\Delta t = 0.05$ (the corresponding dimensionless frequency of energy supply was $\omega = 20$) and with the value of the supplied energy $\Delta E = 0.001$. The free-stream conditions correspond to flight

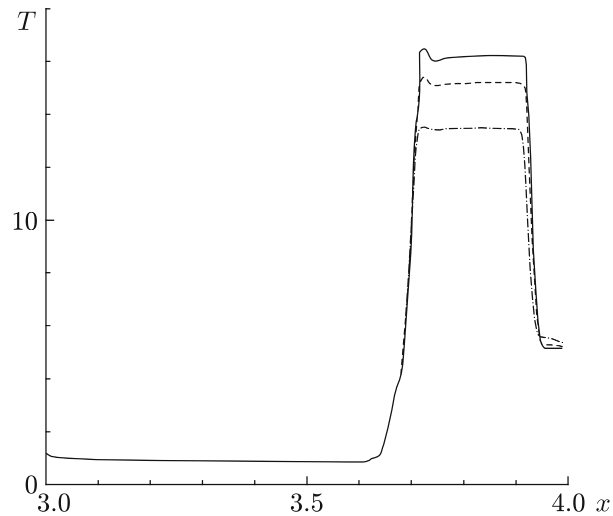

Fig. 1. Temperature distribution along the airfoil: the results are calculated by the ideal gas model (solid curve), by the gas model with allowance for excitation of molecular vibrations (dashed curve), and by the model developed in [10] (dot-and-dashed curve).

conditions at an altitude of 10 km. In accordance with the tabulated data for the standard atmosphere, we had $p_0 = 0.2644$ atm and $T_0 = 223.15$ K [17]. For the same position of the energy supply zone $3.609 \leqslant x \leqslant 3.693$ (the airfoil chord was located on the segment $3 \leqslant x \leqslant 4$), the wave drag coefficient of the airfoil predicted by model No. 1 ($C_x = 0.03507$) was slightly greater than that obtained in the ideal gas model ($C_x = 0.03498$).

Similarly, in the case of energy supply proportional to the local gas density in the zone $3.433 \leqslant x \leqslant 3.523$, model No. 1 (with the specific supplied energy $E = 20$, which corresponded to the value $\Delta E \approx 0.001$) predicted the value $C_x = 0.02162$ close to $C_x = 0.02156$ obtained with the use of the ideal gas model.

Thus, allowance for excitation of vibrational motion of air molecules in calculating its thermodynamic properties in the problem considered exerts almost no effect on the wave drag of the airfoil and on gas dynamics of the flow around the airfoil as a whole.

The calculations by model No. 2 were performed for the supplied energy $\Delta E = 0.001$ in the area $3.609 \leqslant x \leqslant 3.693$. In this case, the wave drag coefficient of the airfoil was $C_x = 0.03558$, which was 2% greater than the value predicted by the ideal gas model. Taking into account dissociation also exerts a weak effect on the wave drag of the airfoil, which confirms the validity of the ideal gas model calculations [3–5].

At the same time, taking into account the real thermodynamic properties of air leads to a significant decrease in temperature on the airfoil surface behind the energy supply zone. Figure 1 shows the behavior of temperature along the airfoil contour predicted by the ideal gas model (solid curve), by model No. 1 (dashed curve), and model No. 2 (dot-and-dashed curve). Allowance for excitation of vibrational motion of molecules only leads to a decrease in temperature by more than 200 K. Allowance for dissociation leads to an additional decrease in temperature by 500 K. As a result, the air temperature near the airfoil surface does not exceed 3500 K. The position of the closing shock wave changes insignificantly, which results in a small change in the wave drag of the airfoil.

The results calculated by the boundary layer model with allowance for viscosity are plotted in Fig. 2, which shows the lift-to-drag ratio K of the airfoils as a function of the supplied energy ΔE for different values of the Reynolds number. Additional varied parameters in the calculations are the free-stream Mach number M_∞, the airfoil thickness C, and the position of the energy supply zone. The positions $3.567 \leqslant x \leqslant 3.600$ and $3.838 \leqslant x \leqslant 3.864$ near the lower surface of the airfoil were chosen on the basis of the results obtained in [5], where it was found that energy supply in these zones makes it possible to increase the lift-to-drag ratio of the airfoil without increasing the lift force. An analysis of results is performed for the energy supply zone $3.838 \leqslant x \leqslant 3.864$, because the above-mentioned effect is manifested to the greatest extent there. It should be noted that energy supply almost in all calculation variants also leads to an increase in the lift-to-drag ratio with taking viscosity into account, despite the increase in the airfoil drag due to energy supply (with the wave drag being ignored). The growth of the airfoil drag is caused by the increase in pressure in the energy supply zone, but the wave drag decreases thereby owing to the

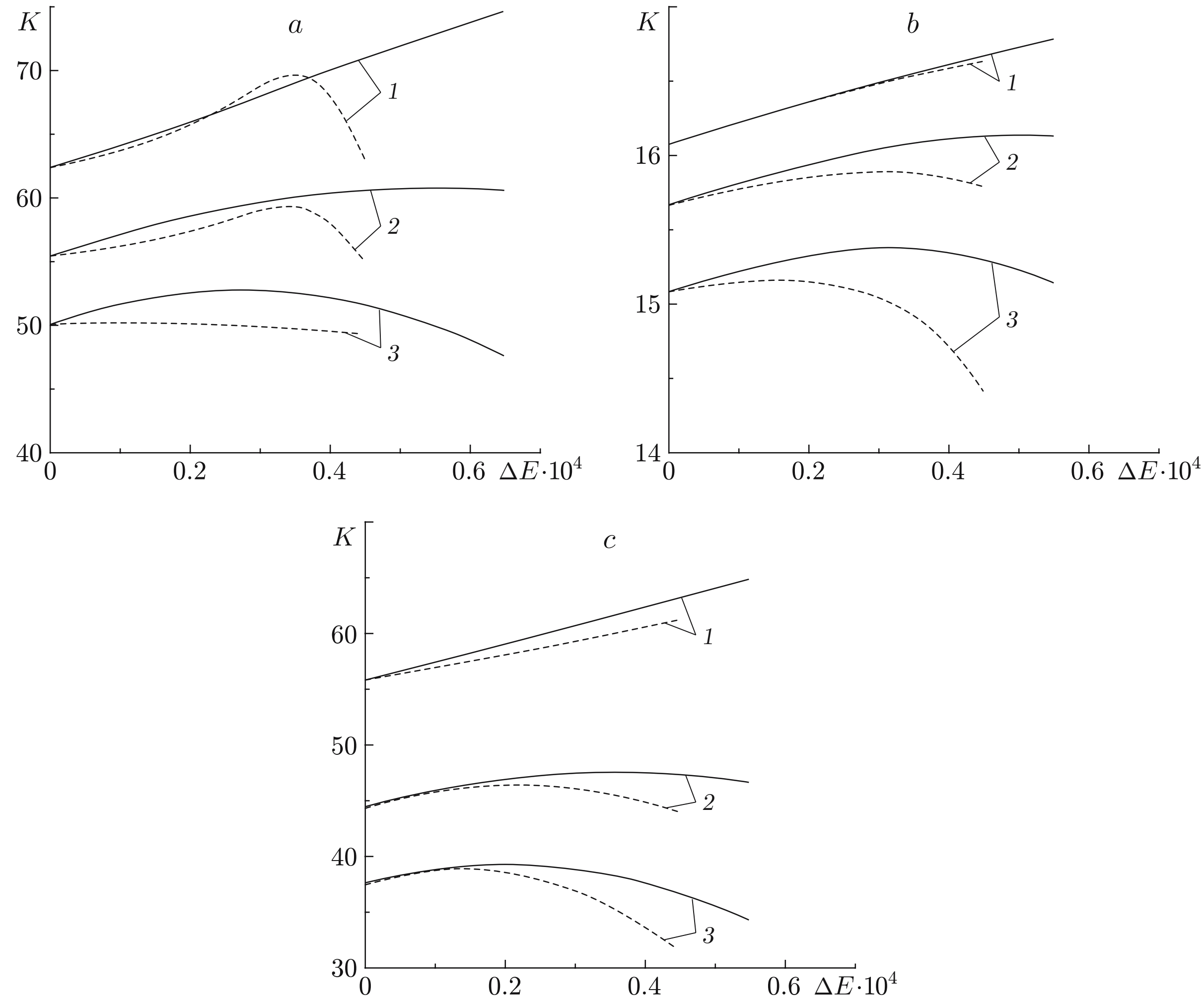

Fig. 2. Lift-to-drag ratio K of the airfoil versus the supplied energy for different Reynolds numbers: (a) $\mathrm{M}_\infty = 0.75$ and $C = 12\%$; (b) $\mathrm{M}_\infty = 0.8$ and $C = 12\%$; (c) $\mathrm{M}_\infty = 0.8$ and $C = 8\%$; $\mathrm{Re} = \infty$ (1), $3 \cdot 10^7$ (2), and $3 \cdot 10^6$ (3); the energy supply area is $3.838 \leqslant x \leqslant 3.864$ (solid curves) and $3.567 \leqslant x \leqslant 3.600$ (dashed curves).

increase in the total pressure on the trailing edge of the airfoil due to energy supply, which decreases the intensity of the closing shock waves. If viscosity is taken into account, a typical feature of the dependence of K on ΔE is the presence of a maximum. Naturally, the relative increase in the lift-to-drag ratio becomes smaller if viscosity is taken into account. For the variant presented in Fig. 2a, the maximum increase in K is 10% at $\mathrm{Re} = 3 \cdot 10^7$ (solid curve 2) and 5.6% at $\mathrm{Re} = 3 \cdot 10^6$ (solid curve 3), whereas the increase in K with ignored viscosity is 17 and 10.5% (solid curve 1) at the energy values corresponding to these maximums. As the free-stream Mach number is increased from $\mathrm{M}_\infty = 0.75$ to $\mathrm{M}_\infty = 0.8$, the fraction of the wave drag in the total drag of the airfoil increases from 83.7% (solid curve 2 in Fig. 2a) to 96.7% (solid curve 2 in Fig. 2b) at $\mathrm{Re} = 3 \cdot 10^7$ and from 76.6% (solid curve 3 in Fig. 2a) to 93% (solid curve 3 in Fig. 2b) at $\mathrm{Re} = 3 \cdot 10^6$. In this case, the relative increase in the lift-to-drag ratio in the case of energy supply with and without allowance for viscosity becomes commensurable: 1.9% at $\mathrm{Re} = 3 \cdot 10^6$, 3% at $\mathrm{Re} = 3 \cdot 10^7$, and 2.8 and 3.5% at $\mathrm{Re} = \infty$. In the calculation variants with allowance for viscosity, the displacement thickness was within $8 \cdot 10^{-4}$ at $\mathrm{Re} = 3 \cdot 10^6$ and $4 \cdot 10^{-4}$ at $\mathrm{Re} = 3 \cdot 10^7$, which confirms the validity of using the results of solving the problem of energy supply within the framework of the inviscid fluid — boundary layer model with subsequent recalculation of aerodynamic characteristics.

A decrease in the airfoil thickness from $C = 12\%$ to $C = 8\%$ (see Figs. 2b and 2c) leads to a significant increase in the absolute value of the lift-to-drag ratio K, whereas the character of the dependence of K on the supplied energy remains unchanged.

The dashed curves in Fig. 2 correspond to the energy supply zone $3.567 \leqslant x \leqslant 3.600$. The effects observed previously (increase in the lift-to-drag ratio K and the maximum on its dependence) are the same, but the values of K are slightly lower.

Conclusions. The study of the influence of pulsed-periodic energy supply on gas-dynamics of a transonic flow around a symmetric airfoil with allowance for real thermodynamic properties of air shows that taking into account these properties exerts almost no effect on the wave drag of the airfoil, but leads to a significant decrease in temperature, as compared with the values predicted by the ideal gas model.

Pulsed-periodic energy supply with allowance for viscosity also allows the lift-to-drag ratio of the airfoil to be increased, though this increase is less significant than in the case of an inviscid flow.

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